Series

1) Consider the series $\sum_{k=0}^{\infty} (-1)^k$

i) Write out the first five partial sums of this series.

ii) Find a general formula for the partial sums. Does it converge? Does the series converge?

2) Consider the series $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$.

i) Does the $n$-th term test say anything about this series?

ii) Show that this is a telescoping series and find a formula for the partial sums. Does the series converge?

3) Consider the series $\sum_{n=0}^{\infty} (1 - \frac{1}{n})^n$. Does the $n$-th term test apply to this series? Does the sequence of general terms converge? Does the series converge?
4) Suppose that \( \sum a_n \) converges. Show that \( \sum \cos(a_n) \) must diverge.

5) For the following series determine whether they converge or not. In case of convergence, find the exact sum.
   i) \( \sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n} \)
   ii) \( \sum_{n=1}^{\infty} \left(-\frac{5}{9}\right)^n \)
   iii) \( \sum_{n=0}^{\infty} \left(\frac{\pi}{6}\right)^n \)

6) Write 0.3 as a geometric series and find its sum

7) Determine whether the following statements are true or false.
   i) If \( \lim a_n = 0 \) then \( \sum a_n \) converges.
   ii) If \( \sum a_n \) converges then \( \lim a_n = 0 \)
   iii) If \( \sum a_n \) does not converge then \( \lim a_n \neq 0 \)
   iv) If \( \sum a_n \) does not converge then who knows what the \( \lim a_n \) is.
   v) A geometric series \( \sum r^n \) is always convergent with sum \( \frac{1}{1-r} \)