Homework 8, Due Monday, Nov 6

This homework must be done individually. Remember to follow Math department's guidelines for homework. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

1. Prove that $A_n \trianglelefteq S_n$. What familiar group is S_n/A_n isomorphic to?

2. Let K be a subgroup of $\langle \mathbb{R}^*, \cdot \rangle$, and let $H = \{A \in GL(2, \mathbb{R}) : \det(A) \in K\}$. Show that $H \trianglelefteq GL(2, \mathbb{R})$.

3. Let $H \trianglelefteq G$ where G is a finite group. Show that the order of an element gH in G/H must divide the order of g in G.

4. Let $H \trianglelefteq G$, $g \in G$ where G is a finite group. If gH has order 4 in G/H and $|H| = 12$, what are the possibilities for $|g|$ in G?

5. An element y is called a square in a group G if $y = x^2$ for some $x \in G$. Suppose G is an abelian group and $H \leq G$. If every element of H is a square, and every element of G/H is a square, prove that every element of G is a square.