Homework on Sections 4 and 5

Due Monday, Sep 11

This homework must be done individually. Remember to follow Math department's guidelines for homework. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged.

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1. Let $G = \mathbb{Q}[\sqrt{2}] := \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$
   (a) Show that G is a group under usual addition of real numbers. Hence $\langle G, + \rangle \leq \langle \mathbb{R}, + \rangle$
   (b) Show that $\langle G^*, \cdot \rangle$ is a group where $G^* = G - \{0\}$ is the non-zero elements of G and $\cdot$ is the usual multiplication of real numbers. Hence $\langle G^*, \cdot \rangle \leq \langle \mathbb{R}^*, \cdot \rangle$
2. Prove that if $\langle G, * \rangle$ is a group such that $x * x = e$ for every $x \in G$, then G is an abelian (commutative) group. Is the converse of this statement true as well?
3. Given $(a, b) \in \mathbb{R}^2$, define $T_{a,b} : \mathbb{R}^2 \to \mathbb{R}^2$ by $T_{a,b}(x, y) = (x + a, y + b)$. Consider $G := \{T_{a,b} : a, b \in \mathbb{R}\}$ with the binary operation $\circ$ of function composition. Is $\langle G, \circ \rangle$ a group? Is it a commutative group? What is the geometric interpretation of the map $T_{a,b}$? What is the geometric explanation for $\circ$ being commutative or not?
4. Do problem 53 in section 5 of the textbook (page 58).