Big-O Notation and Analysis of Algorithms

Having a general idea about the running time of an algorithm is very important for both programmers and the users. Big-O notation is designed to capture the worst-case running time of an algorithm as a function of the size of the input.

Definition: Big-Oh Notation

Let $f, g : \mathbb{N} \to \mathbb{R}^+$. We say that f is "big-oh" of g, written $f = \mathcal{O}(g)$, or $f \in \mathcal{O}(g)$, if ....

Remark 1: A useful way of determining big-O of a function:

Remark 2: The big-O notation is not sensitive to multiplicative constants, lower order terms, or the basis of a logarithm.

Example: a) $f(n) = 2n^3 + 3n^2 + 100$ b) $f(n) = n + 10\sqrt{n} + \log(n)$ c) $f(n) = 2^n + n^7 + 10^3$

Question: Suppose $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$. Is it true that $f(n)$ is $O(h(n))$?

Question: What is $O(1)$? What is $O(n)$?

Example 1: What is the best-case, worst-case and average case running time of the sequential search algorithm?

Example 2: What is the best-case, worst-case and average case running time of the binary search algorithm?

Example 3: What is the number of steps to solve the towers of Hanoi puzzle?

Example 4: What is the running time of the bubble sort algorithm? Is there any difference between the best-case and worst case?

for i $\in \{1, 2, 3, ..., n-1\}$ do
    for j $\in \{1, ..., n-i\}$ do
        if($x_j > x_{j+1}$) then swap(x_j, x_{j+1})

Example 5: Matrix multiplication. The following code multiplies two $n \times n$ matrices A and B, and stores the result in another matrix C. Determine its running time in Big-Oh notation.

```
void matrixmult(int n, const int A[][n],const int B[][n], int C[][n])
{
 int i,j,k;
 for( i=1; i<=n; i++){
   for( j=1; j<=n;j++){
      C[i][j]=0;
      for( k=1; k<=n;k++)
         C[i][j]=C[i][j]+A[i][k]*B[k][j];}}
}
```

Polynomial Time Algorithms: An algorithm is called a polynomial time algorithm if

Size of the Input and Number Theoretic Algorithms Consider the brute-force algorithm to determine whether a given integer is prime? PRIMES is in P.

Remark: If the input for a number theoretical algorithm is integer n, then the size of the input is taken to be .... which is ....

Example: Computational Complexity of Addition, Multiplication and Division