The Solar and Lunar Theory of Ibn ash-Shāṭir: A Pre-Copernican Copernican Model

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The Solar and Lunar Theory of Ibn ash-Shāṭir

A Pre-Copernican Copernican Model

By Victor Roberts *

1. Introduction.

THE purpose of this note is to present the solar and lunar theory developed by the Damascene astronomer, Ibn ash-Shāṭir (1304–1375/6 A.D.) [1] in his *Kitāb Nihāyat as-Sūl fī Taṣḥīḥ al-Uṣūl* (A Text of Final Inquiry in Amending the Elements). Since he dispenses completely with the Ptolemaic eccentric deferent and introduces a second epicycle, both his solar and lunar models are non-Ptolemaic. What is of most interest, however, is that his lunar theory, except for trivial differences in parameters, is identical with that of Copernicus [2] (1473–1543).

Ptolemy assumed a circular path for the sun, whereas the orbit of Ibn ash-Shāṭir's sun deviates slightly from circular motion. The major fault of the Ptolemaic lunar model is in its exaggeration of the variation in lunar distance. The major Copernican contribution to the lunar theory lies in the elimination of this Ptolemaic fault.

Five copies of Ibn ash-Shāṭir's work are known to exist, four of which are at the Bodleian Library. They are MSS Marsh 139, Marsh 290, Marsh 501, and Hunt 547. The fifth copy is at Leiden, Arabic MS 1116.[3]

In the preparation of this paper a microfilm of the first of the above-mentioned copies was used, made available through the courtesy of the Keeper of Oriental Books at the Bodleian. This manuscript of sixty-four folios was copied in 768 A.H. (1366 A.D.), i.e., within the lifetime of the author. In the introduction Ibn ash-Shāṭir claims that in his treatise he is presenting a planetary theory of finest achievement, and that he has traced and explained all astronomical uncertainties in his *Ta'līq al-Arṣād* (Comments on Observations). Unfortunately, this work is apparently non-extant.

According to our source the inventor's name is Abū'l-Ḥasan 'Alī ibn Ibrāhīm ibn Maḥmalain al-Humām ibn Muḥammad ibn Ibrāhīm 'Abdur-Raḥmān al-Anṣārī. Other versions are: Abū'l-Ḥasan 'Alī bin Ibrāhīm bin Muḥammad al-Muṭ'im al-Anṣāri,[4] and 'Alā' ud-Dīn 'Alī bin Ibrāhīm Muḥammad.[5] In all

* The American University of Beirut.

[1] Suter, H., *Die Mathematiker und Astronomen der Araber und ihre Werke*, Abhandlungen zur Geschichte der Mathematik . . . (Leipzig, 1900), X. Heft, p. 168.

[2] This was pointed out by Professor O. Neugebauer.

[3] Brockelmann, C., *Geschichte der arabischen Litteratur* (2d ed., Leiden, 1943), vol. 2, p. 156.

[4] Suter, *op. cit.*, p. 168.

[5] Brockelmann, *loc. cit.*

versions he is known by the name of Ibn ash-Shāṭir. Besides being an astronomer, he held the post of *muwaqqit* — one who determines the time of prayer — at the Umayyad Mosque in Damascus.

2. *The Solar Theory.*

The mean sun is represented by the center a (Figure 1) of an epicycle moving ing from west to east above the horizon with a daily mean motion of 0;59,8,9,51,-46,57,32,3° [6] around the center of the universe E on a deferent of radius 1,0;0.

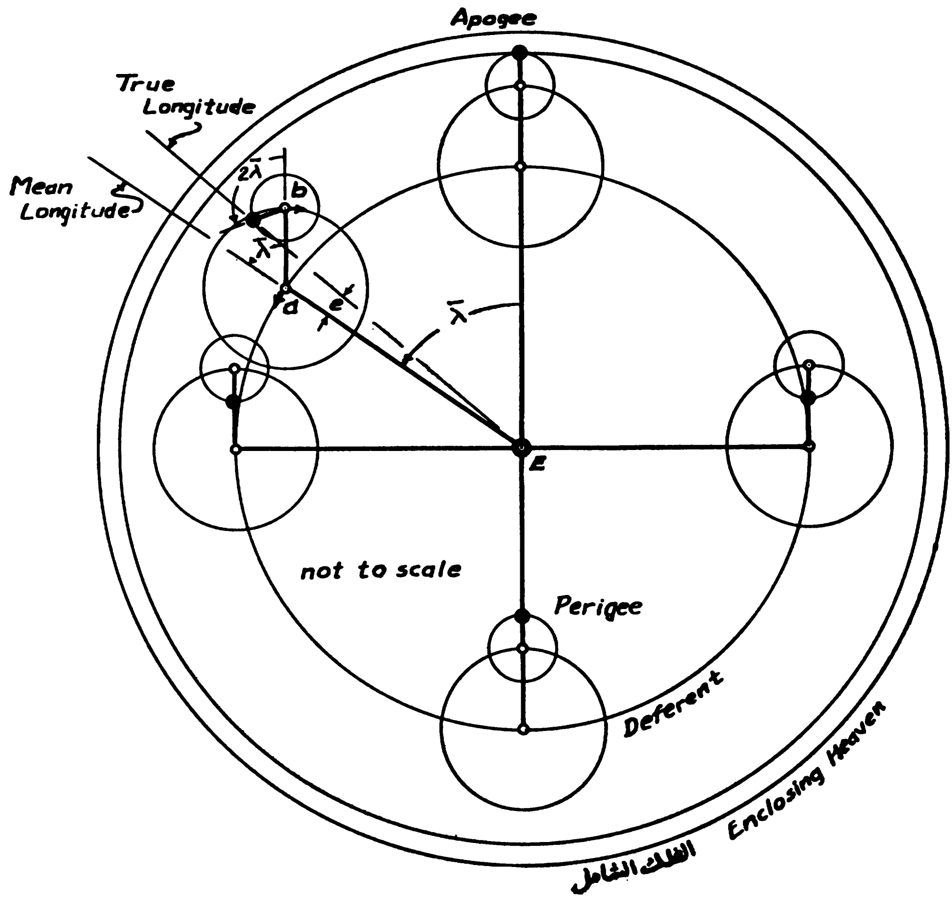

FIG. 1.

The radius of this epicycle, in length 4;37 units, rotates with respect to the deferent radius, at the same speed but in the opposite sense, carrying the center b of a second epicycle of radius 2;30. This latter epicycle, with the true sun of radius 0;10 on it, rotates with respect to the first epicycle radius

[6] The value is expressed in the sexagesimal system. For instance, in our notation 1,2;32,59, stands for $1\cdot 60^1 + 2\cdot 60^0 + 32\cdot 60^{-1} + 59\cdot 60^{-2}$.

at double the daily mean motion and in the same direction. The ***enclosing heaven*** (*al-falak ash-shāmil*), has an inner radius equal to the sum of the three given radii, or 1,7;17. Its width is 0;43, apparently in order to round off its outer radius to an integer (1,7;17 + 0;43 = 1,8;0). The enclosing heaven moves from west to east at the rate of 0;0,0,9,51,46,51° per day. This is the motion of the solar apogee, and comes to one degree in sixty Egyptian years of 365 days each. Figure 1 shows the initial and subsequent positions of the true sun. With the given values of the parameters the solar distance varies between 52;53 and 1,7;7. Accordingly, the apparent solar diameter has a mean value of 0;32,32°, varying between extreme values of 0;29,5° and 0;36,55°.

The maximum solar equation e given by Ibn ash-Shāṭir is 2;2,6°, and occurs at a mean longitude $\bar{\lambda}$ of 97° or 263°, measured from apogee. It disappears at the apogee and perigee. These values have been verified by independent computation by the present author. The following method, expressed in modern symbolism, is given in the manuscript for computing e and the solar distance, ρ.

$$7;7 \cos \bar{\lambda} = \mathrm{A}, \text{ say,}$$

(The manuscript has $\sin \bar{\lambda}$, an obvious error.)

$$1,0;0 + \mathrm{A} = \mathrm{B}, \text{ say}$$

(The manuscript says add A if $97°\ 7' > \bar{\lambda} > 262°\ 53'$, otherwise, subtract A. Such special rules to take care of negative functional values are typical of mathematical exposition prior to the use of signed numbers. The limits of $\bar{\lambda}$ represent the mean longitudes which yield the mean solar distance.)

$$2;7 \sin \bar{\lambda} = \mathrm{c}, \text{ say,}$$

therefore $$\rho = \sqrt{\mathrm{B}^2 + \mathrm{c}^2},$$

and $$\sin \mathrm{e} = \frac{\mathrm{c}}{\rho}.$$

With simple geometry this method can easily be verified by noting that 7;7 and 2;7 are respectively the sum and difference of the radii of the epicycles.

The only observation mentioned in this connection is one made in Damascus on Tuesday, the first day of the year 701 of Yazdigerd, or 24 Rabī' al-Awwal 732 A.H. (25 December, 1331). On the mid-day of this date, according to the inventor, the mean sun and its apogee showed longitudes of 5,9;0° and 1,19;12° respectively.

Ibn ash-Shāṭir gives no motivation for his adoption of two epicycles, and it is difficult to see how his model constitutes an improvement over the Ptolemaic one.

3. *The Lunar Theory.*

The orbit of the moon is inclined at an angle of 5° to the parecliptic [7] (of radius 1,9;0), and moves from east to west at the rate of 0;3,10,38,27° per day.

[7] The parecliptic (*al-falak al-mumaththal*) is defined as a circle in the plane of the ecliptic with the earth as its center.

The deferent, of radius 1,0;0, moves from west to east around the center of the universe (*R* in Figure 2) at the daily mean latitudinal rate of 13;13,45,39,40°. Therefore, the mean value of the moon is the sidereal motion, or 13;10,35,1,13°, which is the difference between the two values given above. The first epicycle radius *ab*, in length 6;35, rotates with respect to *Ra* at a daily anomalistic rate *a* of 13;3,53,46,18°, and in a direction opposite to the mean motion. To explain the "second inequality" of the moon, Ibn ash-Shāṭir places the true moon on a second epicycle *bc* of radius 1;25 whose daily motion from west to east is equal to double the elongation ϵ or 24;22,53,23° (double the difference between the mean sun 0;59,8,20°, and the mean moon). It is of interest to compare the radii of the epicycles with the Copernican values, which are 6;34,55,12 and 1;25,19, 12 [8] respectively. Figure 2 illustrates the motion of the moon, starting from a mean conjunction in which *R*, *a*, and *b* are collinear.

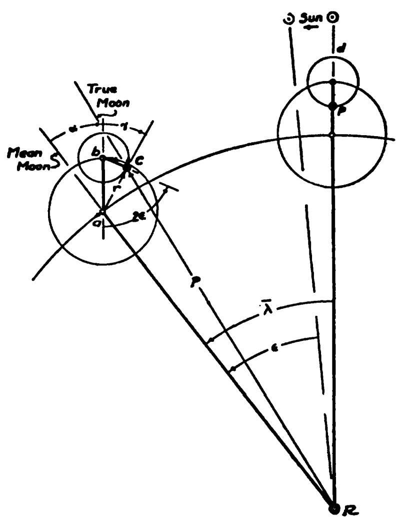

FIG. 2.

As a consequence of the resultant motion, the moon will always be in perigee *p* of the second epicycle at mean syzygies, and in its apogee *d* at quadratures. Hence, the apparent epicycle of radius 5;10 (difference of the epicyclic radii) at the syzygies accounts for the equation of the center, and the gradual increase in its apparent radius (maximum 8;0) as it approaches the quadratures accounts for the evection. The maximum values of the first and second inequalities are 4;56 and 2;44 respectively. Thus Ibn ash-Shāṭir has retained the ancient sum of the two inequalities, which is obtained at the quadratures when the line from the earth *R* to the true moon is tangent to the apparent epicycle.

It follows from the given values of the parameters that the lunar distance

[8] *De revolutionibus*, IV, 8. See also Dreyer, J. L. E., *History of the Planetary Systems* (Cambridge, 1906; republished in 1953 under the title, *A History of Astronomy from Thales to Kepler*), p. 336.

varies between 54;50 and 1,5;10 at the syzygies, and between 52;0 and 1,8;0 at the quadratures. Accordingly, the apparent lunar diameter, with a mean value of 0;32,54,33°, varies between extremes of 0;29,2,15 and 0;37,58,20°. (The last two values are computed from the first, the only one given in the text.)

The manuscript gives the following method, expressed in modern symbolism, for computing e, the lunar equation.

$$\begin{aligned} 1;25 \sin 2\epsilon &= \mathrm{A}, \text{ say,} \\ 1;25 \cos 2\epsilon &= \mathrm{B}, \text{ say,} \\ 6;35 + \mathrm{B} &= \mathrm{C}, \text{ say.} \end{aligned}$$

(The manuscript says, subtract B if $90° \geqq 2\epsilon \geqq 0°$ or $360° \geqq 2\epsilon \geqq 270°$, otherwise add.)

$$\sqrt{\mathrm{C}^2 + \mathrm{A}^2} = \mathrm{r}, \text{ the radius of the apparent epicycle.}$$

Therefore $\sin \gamma = \frac{\mathrm{A}}{\mathrm{r}}$, or $\gamma = \sin^{-1} \frac{\mathrm{A}}{\mathrm{r}}$.

$$\begin{aligned} 5;10 \sin (\alpha + \gamma) &= \mathrm{D}, \text{ say,} \\ 5;10 \cos (\alpha + \gamma) &= \mathrm{E}, \text{ say,} \\ 1,0;0 + \mathrm{E} &= \mathrm{F}, \text{ say.} \end{aligned}$$

(Add E if $90° \geqq (\alpha + \gamma) \geqq 0°$ or $360° \geqq (\alpha + \gamma) \geqq 270°$, otherwise, subtract E.)

$$\sqrt{\mathrm{D}^2 + \mathrm{F}^2} = \rho.$$

Therefore $\sin e = \frac{\mathrm{D}}{\rho}$, where e is the first inequality. When 5;10 is replaced by 8;0, e will represent the sum of the two inequalities.

4. *Concluding Remarks.*

The same manuscript gives full parameters and descriptions of the models of the other planets. These await further study. Suffice it to say at the present that these, like the solar model, are quite different from those of Copernicus.

The question which immediately arises is that of possible influence of the earlier astronomer on the later. There seems to be no way at present of resolving the problem either in one way or the other.

Ibn ash-Shāṭir's work was fairly well-known in the Near East, as numerous copies and recensions of his astronomical handbook (*Az-Zīj al-Jadīd*) and other works testify.[9] There is no evidence for any of his books having been translated into Latin,[10] and it is possible that the two individuals hit upon the same solution independently. On the other hand, Copernicus utilized the work of other Islamic scientists, notably that of Thābit bin Qurra. *Allāhu a'lim* (God knows better).

[9] Dr. D. J. Price suggests that Ibn ash-Shāṭir may have been the maker of two astrolabes preserved in Paris collections, Nos. 6 and 142 (in Gunther, *The Astrolabes of the World*, Oxford, 1932).

[10] For instance, Ibn ash-Shāṭir is not mentioned by F. J. Carmody in his *Arabic Astronomical and Astrological Sciences in Latin Translation. A Critical Bibliography*, University of California Press, 1956.