A Quick Review

1. What is the most important concept/notion of Calculus?

2. Let f be a function such that $f(x) \geq 0$ for $a \leq x \leq b$. What is the geometric meaning of $\displaystyle\int_a^b f(x)dx$? What if $f(x) \leq 0$ for $a \leq x \leq b$?

3. How is $\displaystyle\int_a^b f(x)dx$ formally defined?

4. Compute the derivative of the functions $\sqrt{x} \cdot e^{2x}$ and $\ln(\sin(x^2))$

5. State the two versions of the fundamental theorem of Calculus.

6. Compute the following integrals: $\displaystyle\int_1^e \frac{1}{x}dx$ and $\displaystyle\int_0^1 e^{2t}dt.$

Section 5.5: Substitution Rule

Recall that FTC part II gives us a convenient way to evaluate a definite integral: $\displaystyle\int_a^b f(x) = F(x)\Big|_a^b$. However, we still have the task of finding an antiderivative for a given function. That could be easy, harder, very difficult, or even impossible (and anything in between). In general finding antiderivatives is harder than finding derivatives. Some integrals can be found using elementary formulas that we have so far, but others require more involved techniques. There are a number of techniques to compute antiderivatives, and most of them are covered in Calc II. In this course, we'll see one basic method, the method of substitution.

The Method Of Substitution

Remark: Every differentiation formula has a corresponding integration (anti-differentiation) formula. If $F'(x) = f(x)$, then what is the corresponding integration formula?

The method of substitution is the opposite of the chain rule. For example, consider the derivative $\sqrt{\sin x}$. What is this derivative? What rule of differentiation have you used? What is the corresponding integration formula?

More generally, the chain rule says $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$. So, the corresponding indefinite integral formula is ...

Now, try the following examples

A: Basic Substitutions

(1) $\displaystyle\int 2xe^{x^2}\,dx$

(2) $\displaystyle\int x\sqrt{x^2+1}dx$

(3) $\displaystyle\int \frac{1}{2x+1}dx$

(4) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

(5) $\displaystyle\int \frac{dx}{\sqrt{5-3x}}$

(6) $\displaystyle\int \frac{e^x}{e^x+1}dx$

B: Definite Integrals and Substitution

(1) $\displaystyle\int_0^{\pi/3} \frac{\sin(x)}{\cos^3(x)}dx$

(2) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(3) $\displaystyle\int_0^1 \frac{\arcsin(x)}{\sqrt{1-x^2}}dx$

C: More Complicated Substitutions

(1) $\displaystyle\int \frac{x}{1+x^4}dx$

(2) $\displaystyle\int x^5\sqrt{1+x^2}dx$

(3) $\displaystyle\int \sqrt{1+2\sqrt{x}}dx$

(4) $\displaystyle\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ (Hint: Try $u=\sqrt[6]{x}$)

Some Guidelines:

How To Choose u

- It is usually a quantity that is inside of $\sqrt{\cdots}$, $e^{\cdots}$, $(\ldots)^a$, $\frac{1}{(\ldots.)}$, $\sin(\ldots.)$ etc.
- Derivative of u, up to a constant multiple, should be around somewhere in the integrand
- We can always adjust for multiplicative constants

General Remarks

- After a u-substitution, everything must be expressed in terms of u and du.
- $\int f(x)dx \to \int g(u)du$
- Cannot have a mixture of u and x.
- After integrating $\int g(u)du$, and getting something of the form $G(u)$, you must substitute u back so that the final answer is a function of x.
- If you have a definite integral, then change the limits for u and you do not need to go back to x after integration.