1) (10 points each) Evaluate the following:

a) \( \int \cot(x) \, dx \)

**Solution:** \( u = \sin(x) \), \( du = \cos(x) \, dx \)

\[
\int \cot(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx = \int \frac{1}{u} \, du = \ln |u| + C = \ln |\sin(x)| + C
\]

b) \( \int \frac{1}{\sqrt{x}(1 + \sqrt{x})} \, dx \)

**Solution:** \( u = 1 + \sqrt{x} \), \( du = \frac{1}{\sqrt{x}} \, dx \)

\[
\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} \, dx = 2 \int \frac{1}{u} \, du = 2 \ln |u| + C = 2 \ln |1 + \sqrt{x}| + C
\]
c) \[ \int \frac{x}{\sqrt{4-x}} \, dx \]

Solution: \( u = 4 - x \), \( du = -dx \) \( x = 4 - u \)

\[
\int \frac{x}{\sqrt{4-x}} \, dx = - \int \frac{4-u}{\sqrt{u}} \, du \\
= - \int (4u^{\frac{1}{2}} - u^{\frac{3}{2}}) \, du \\
= -(8u^{\frac{3}{2}} - \frac{2}{3}u^{\frac{5}{2}}) + C \\
= -8\sqrt{4-x} - \frac{2}{3}(4-x)^{\frac{3}{2}} + C
\]

d) \[ \int \frac{2x}{e^x} \, dx \]

Solution: \( u = 2x \), \( du = 2 \, dx \), \( dv = e^{-x} \, dx \), \( v = -e^{-x} \)

\[
\int \frac{2x}{e^x} \, dx = -2xe^{-x} + 2 \int e^{-x} \, dx \\
= -2xe^{-x} - 2e^{-x} + C
\]
e) $\int \frac{(\ln(x))^2}{x} \, dx$

Solution: $u = \ln(x), \; du = \frac{1}{x} \, dx$

\[
\int \frac{(\ln(x))^2}{x} \, dx = \int u^2 \, du = \frac{u^3}{3} + C = \frac{(\ln(x))^3}{3} + C
\]

2) (10 points) A 20-foot chain weighing 8 pounds per foot is lying on the ground. How much work is required to raise the chain 20-feet so that it is fully extended vertically.

Solution: Weight: 8 Deltay, Distance Lifted : $y$

\[
\int_0^{20} 8y \, dy = 4y^2\bigg|_0^{20} = 1600
\]
3) (10 points) Find the volume of the solid of revolution generated by revolving the region bounded by \( y = 6 - x^2 \), and \( y = 2 \) about the line \( y = 2 \).

**Solution:** \( 6 - x^2 = 2 \Rightarrow x = \pm 2 \), radius = \( 6 - x^2 - 2 = 4 - x^2 \)

\[
\pi \int_{-2}^{2} (4 - x^2)^2 \, dx = \pi \int_{-2}^{2} (16 - 8x^2 + x^4) \, dx
\]

\[
= \pi \left( 16x - \frac{8x^3}{3} + \frac{x^5}{5} \right)_{-2}^{2}
\]

\[
\approx 34.13\pi
\]

4) (10 points) The base of a certain solid has the shape of the region bounded by \( y = x^2 \), \( y = -x^2 - 1 \), \( x = 0 \), and \( x = 2 \). Determine the volume of the solid if vertical cross sections perpendicular to the \( x \)-axis are semicircles.

**Solution:** Diameter = \( x^2 - (-x^2 - 1) \) \( \Rightarrow \) Radius = \( \frac{2x^2 + 1}{2} \) \( \Rightarrow \) \( A(x) = \frac{\pi}{2} \left( \frac{2x^2 + 1}{2} \right) = \frac{\pi}{8} (4x^4 + 4x^2 + 1) \)

\[
\frac{\pi}{8} \int_{0}^{2} (4x^4 + 4x^2 + 1) \, dx = \frac{\pi}{8} \left( \frac{4x^5}{5} + \frac{4x^3}{3} + x \right)_{0}^{2} \approx \frac{38.27\pi}{8}
\]
5) (10 points) Given the initial value problem

\[
\begin{align*}
\frac{dy}{dx} &= x - y^2 \\
y(0) &= 1
\end{align*}
\]

approximate \(y(1)\) using Euler’s method with step size equal to 0.25.

**Solution:**
\[
\begin{align*}
x_0 &= 0, \quad y_0 = 1 \\
x_1 &= 0.25, \quad y_1 = 1 + 0.25(0 - 1^2) = 0.75 \\
x_2 &= 0.5, \quad y_1 = 0.75 + 0.25(0.25 - (0.75)^2) = 0.671875 \\
x_3 &= 0.75, \quad y_1 = 0.671875 + 0.25(0.5 - (0.671875)^2) = 0.684021 \\
x_4 &= 1, \quad y_1 = 0.684021 + 0.25(0.75 - (0.684021)^2) = 0.75455
\end{align*}
\]

6) (10 points) Given the slope field for the differential equation \(\frac{dy}{dx} = x - y^2\), draw an approximate solution to the initial value problem given in question (5).

**Solution:**

![Slope Field Image]