Math. 152 – Midterm 1 – Form A

SHOW YOUR WORK!

Note: A random sample of graded exams will be xeroxed.

1) (15 points) The following sum

\[
\left| 3 - \frac{5}{n} \right| \frac{5}{n} + \left| 3 - \frac{10}{n} \right| \frac{5}{n} + \left| 3 - \frac{15}{n} \right| \frac{5}{n} + \ldots + \left| 3 - \frac{5n}{n} \right| \frac{5}{n}
\]

is a right Riemann sum approximating a definite integral \( \int_{a}^{b} f(x) \, dx \), using a partition of the given interval \([a, b]\) into \(n\) equal parts. Write down such a definite integral. Using this fact compute the limit of these sums as \(n \to \infty\). (Note: you can interpret the integral as the sum of areas of two elementary geometric figures.)

2) (26 points) Find the following indefinite integrals

(a) \( \int \frac{5t^2 - 2}{t^2} \, dt \)  
(b) \( \int \frac{\cos(6x)}{\sin(6x) + \pi} \, dx \)
3) (26 points) Find the following definite integrals

(a) \[ \int_{-1}^{1} \frac{1}{2 + |u|} \, du \]  
(b) \[ \int_{0}^{\pi/3} \frac{\sin(x)}{\cos^6(x)} \, dx \]

4) (15 points) Find the derivative \( \frac{dy}{dx} \) where \( y = \int_{0}^{\sqrt{x}} s \sqrt{1 + s^4} \, ds \), using the Fundamental Theorem of Calculus.
5) (18 points) Find the total area between the graphs of \( y = x^3 - 3x \) and \( y = x^2 - x \). Note that the enclosed region consists of two parts.