

Problem 5: Linear Algebra over Finite Fields

Let \mathbb{F}_q denote the finite field with q elements, where q is a prime power, and let V be a vector space of dimension n over \mathbb{F}_q .

1. Determine the number of vectors in V .
2. Let $GL_n(\mathbb{F}_q)$ denote the set of $n \times n$ non-singular (invertible) matrices over \mathbb{F}_q . Determine the size of $GL_n(\mathbb{F}_q)$.
3. Let $SL_n(\mathbb{F}_q)$ denote the set of $n \times n$ matrices over \mathbb{F}_q with determinant 1. Determine the size of $SL_n(\mathbb{F}_q)$.

As always, show your work, fully explain and justify your answer.

Posting Date 3/19/2017. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 3/31/17.