Problem of the Week-4: A Piecewise Defined Function

Consider the function $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

1. Is f continuous at $x = 0$?
2. Is f differentiable at $x = 0$?
3. Is f' continuous at $x = 0$?
4. Is f' differentiable at $x = 0$?

As always, show your work, fully explain and justify your answers.

Posting Date 2/18/15. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 5 pm on 2/27/15.