Problem of the Week 4: Fibanocci on the Hyperbola

Let $f_0 = 1, f_1 = 1$ and $f_k = f_{k-1} + f_{k-2}$ be the Fibanocci numbers. Show that

1. the points (f_k, f_{k+1}) for all $k \geq 1$ lie on one of the hyperbolas $y^2 - xy - x^2 = \pm 1$
2. the only points on the hyperbolas $y^2 - xy - x^2 = \pm 1$ with non-negative integer coefficients are the points in part 1. [1]

[1] Posted: 2/24/05 Submit your answers (by e-mail or hard copy) before 4 pm on 3/4/05 to Noah Aydin, Mathematics Dept. If you can only prove part 1 and nobody else can prove both parts, then you will win the prize!