Problem 2: Triangular Numbers

Triangular numbers are so named because they can be arranged in a triangular configuration. The first few triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, ...  
Consider the following pattern

\begin{align*}
1 &= 1 \\
1 + 5 &= 6 \\
1 + 7 + 7 &= 15 \\
1 + 9 + 9 + 9 &= 28 \\
1 + 11 + 11 + 11 + 11 &= 45 \\
&\vdots
\end{align*}

This is a pattern that gives every other triangular number (the first, third, fifth, ...) Find a closed formula for the $n^{th}$ term of this sequence. Then find a similar pattern for even indexed triangular numbers (second, fourth, sixth, ...) and find a closed formula for the $n^{th}$ term of that sequence.

As always, show your work, fully explain and justify your answer.