**Solution to Problem of the Week 4**

$$
\begin{aligned}
\sum_{i=0}^{n}\sum_{j=0}^{n-i}\frac{x^j}{i!j!} &= \sum_{i=0}^{n}\sum_{k=i}^{n}\frac{x^{k-i}}{i!(k-i)!} && \text{Let } k=i+j \\
&= \sum_{i=0}^{n}\sum_{k=i}^{n}\frac{1}{k!}\binom{k}{i}x^{k-i} \\
&= \sum_{i=0}^{n}\sum_{k=0}^{n}\frac{1}{k!}\binom{k}{i}x^{k-i} && \binom{k}{i}=0 \text{ if } k<i \\
&= \sum_{k=0}^{n}\frac{1}{k!}\sum_{i=0}^{n}\binom{k}{i}x^{k-i} && \text{interchange the summations} \\
&= \sum_{k=0}^{n}\frac{1}{k!}\sum_{i=0}^{k}\binom{k}{i}x^{k-i} && \binom{k}{i}=0 \text{ if } i>k \\
&= \sum_{k=0}^{n}\frac{1}{k!}(1+x)^k && \text{binomial expansion of } (1+x)^k
\end{aligned}
$$

Finally, as $n \to \infty$ the last sum converges to $e^{1+x}$, for any $x$, by the power series expansion of the exponential function.