Show that among any set of 15 positive integers $n, 1 < n \leq 2023$, that are mutually coprime, there is at least one prime.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 9/8/2023. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Friday, September 22, 2023. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.