Problem 7: Fibonacci Sequence \( \mod N \)

Let \( N > 3 \) be a positive integer and consider the Fibonacci sequence \( \mod N \), that is,

\[
a_1 = a_2 = 1, \quad a_{n+1} = (a_n + a_{n-1}) \mod N \quad \text{for} \quad n \geq 2.
\]

Prove that the sequence \( a_1, a_2, a_3, \ldots \) is periodic. What is the maximum possible length of the period?

As always, show your work, fully explain and justify your answer.