

Problem of the Week-4: Convergent or Divergent?

Let that $a_n = a_1, a_2, a_3, \dots$ be a sequence of positive real numbers such that $\sum_{n=1}^{\infty} a_n$ diverges and let $s_n = a_1 + a_2 + \dots + a_n$ then show that:

1.

$$\sum_{n=1}^{\infty} \frac{a_n}{s_n} \text{ diverges,}$$

2.

$$\sum_{n=1}^{\infty} \frac{a_n}{s_n^2} \text{ converges.}$$

As always, explain and justify your answer.

Hints:

1. Show that $\frac{a_{N+1}}{s_{N+1}} + \dots + \frac{a_{N+k}}{s_{N+k}} \geq 1 - \frac{s_N}{s_{N+k}},$

2. Show that $\frac{a_n}{(s_n)^2} \leq \frac{1}{s_{n-1}} - \frac{1}{s_n}$

Posting Date 10/08/11. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 10/21/11.