Problem of the Week-2: Rational or Irrational?

Determine whether the following statements are True or False. Provide a thorough justification for each.

1. The product of a rational number with an irrational number is irrational.

2. The product of a non-zero rational number with an irrational number is irrational.

3. The sum of an irrational number and a rational number is irrational.

4. The sum of two irrational numbers is irrational

5. $\sqrt[3]{3}$ is irrational.

As always, explain and justify your answer.